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Source / Izvornik: **Croatian Journal of Philosophy, 2019, 19, 597 - 610**

Journal article, Published version

Rad u časopisu, Objavljena verzija rada (izdavačev PDF)

Permanent link / Trajna poveznica: <https://um.nsk.hr/um:nbn:hr:261:503349>

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Download date / Datum preuzimanja: **2025-03-14**



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Identity between Semantics and Metaphysics

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In this paper, I consider several issues related to the concept of identity—the concept that is in many ways related to Heda Festini's early philosophical interests. I specifically focus on discussion of the issues in Frege, Russell, and Wittgenstein. I contrast two competing conceptions of identity—the objectual (according to which identity is a relation in which every object stands only to itself) and the metalinguistic (according to which identity is a relation between coreferential names)—and consider reasons these authors had for accepting or discarding one or the other. In addition, I consider how issues concerning identity relate to issues concerning identity statements.

Keywords: Frege, identity, identity statements, indiscernibility of identicals, informativeness, metalinguistic conception, objectual conception, relation, Russell, Wittgenstein.

1.

In 1992 Heda Festini published a book *Uvod u čitanje Ludwiga Wittgensteina* [*An Introduction to Reading Ludwig Wittgenstein*], covering in an introductory yet novel way major themes in Wittgenstein's philosophical development, from early *Notebooks* to his late *On Certainty*. In the course of that, she paid a particular attention to issues that occupied her own thinking over the two previous decades, from mid 1970ties onward. The first of the issues concerns the connection between Wittgenstein's analysis of language-games and linguistic meaning as based on use, and Dummett's and Hintikka's semantic conceptions, as well as their antirealist inclinations (see e.g. Festini 1985, 1986/1987, 1988/1989). The second one concerns Wittgenstein's earlier semantic insights, and his implicit or explicit exploitation and exploration of Fregean sense/reference (or, more generally, intension/extension).

sion) distinction, particularly in *Tractatus Logico-Philosophicus* and writings of the middle period (see e.g. Festini 1976/1977, 1978, 1982). Festini's book was never intended to be a complete, overall exposition of Wittgenstein's ideas, of course, and a number of his ideas—even those closely related to her primary interests—she never discussed. Among them is Wittgenstein's (2001) criticism of the traditional, "objectual" conception of identity (according to which it is a relation in which every object stands to itself and no other object), and his elimination of the identity sign from conceptual notation (i.e. logic) altogether.

Naturally, the issues concerning identity were not only in Wittgenstein's focus. They were of considerable interest to his predecessors—Frege and Russell—whom early Wittgenstein identified as central figures affecting his thought (Wittgenstein 2001: 4). Indeed, Wittgenstein explicitly identified Russell's (and Whitehead's 1927: 22, 57, 168) definition of identity as the primary target of his criticism (Wittgenstein 2001: 5.5302). But most of his critical remarks concern other related conceptions as well. Russell (2001: xviii) initially thought it is "a destructive criticism from which there seems no escape", but subsequently changed his mind, seeing it instead as "invalid" and "mistaken" (Russell 1959: 115).

The definition of identity Russell and Whitehead proposed in *Principia Mathematica* clearly relates to Frege's views on identity.¹ Indeed, they all fall within the "Leibnizian" tradition that in one way or another exploits the indiscernibility of identicals principle, which Leibniz formulated as: "Things are the same as each other, of which one can be substituted for the other without loss of truth" (Frege 1980a: 76; 1984: 200).² In addition, Wittgenstein's more positively oriented remarks about identity in *Tractatus*—what identity would amount to if it turned out not to be eliminable—bare similarities to Frege's treatment of identity in *Conceptual Notation*. Wittgenstein, for example, writes (2001: 4.241): "When I use two signs with one and the same meaning, I express this by putting the sign '=' between them. / So ' $a = b$ ' means that the sign ' b ' can be substituted for the sign ' a '." Both of them, at the time, would say that identity is a matter of linguistic conventions, rather than a sterile objectual relation. It would be a relation between names of objects provided they are coreferential, rather than objects themselves; call this the "metalinguistic" conception.

¹ The peculiarity of Russell's and Whitehead's definition— $x = y =_{\text{def.}} \text{ 'F (F!x} \rightarrow \text{F!y)}$ —stems from their hierarchisation of functions (generally, drawn to avoid various antinomies), the definition appealing only to the predicative functions. So, they insist: "We cannot state that every function satisfied by x is to be satisfied by y , because x satisfies functions of various orders. And these cannot all be covered by one apparent variable" (1927: 168; see also p. 57). Wittgenstein (2001: 5.5302) pointed to an addition problem with *that* feature of Russell's and Whitehead's definition, but in what follows, I will not consider it further.

² In fact, the tradition would be more accurately labelled "Aristotelian"; see Kneale and Kneale (1962: 42).

The concept of identity was of considerable interest to Frege, and Frege's insights about it made a considerable impact on Russell (and Wittgenstein), as well as on Dummett and Hintikka. In turn, Wittgenstein and the latter two influenced much of Festini's thinking over the two decades. And if one adds to all that that the intensional/ex-tensional distinction is typically defined in terms of the unrestricted substitutivity that stems from identity, the concept of identity seems to be an appropriate theme for a paper included in a collection dedicated to Festini.

2.

In *The Principles of Mathematics*, Russell summarised much of the basic worries surrounding the concept of identity of the period from Frege's *Conceptual Notation* and "On Sense and Reference" to Wittgenstein's *Tractatus*. He writes:

The question whether identity is or is not a relation, and even whether there is such a concept at all, is not easy to answer. For, it may be said, identity cannot be a relation, since, where it is truly asserted, we have only one term, whereas two terms are required for a relation. And indeed identity, an objector may urge, cannot be anything at all: two terms plainly are not identical, and one term cannot be, for what is it identical with? Nevertheless identity must be something. (Russell 1992: 63)

Here, as in many other related passages of that period, the worry starts as a metaphysical one. Russell asks, does identity *exist*, and, if it does, what is its *nature*. Immediately, however, the discussion becomes a *semantic* one—the focus now being on “where it [identity] is truly asserted” rather than on identity itself. The reason is obvious. If one limits himself strictly to metaphysical issues, the claims with which one end up are either largely uninformative, trivial, and impotent, or plainly contradictory, or even nonsensical. Namely, all one can say is that identity is a relation in which every object stands to itself and no other object, and then specify properties of that relation, such as reflexivity, symmetry, and transitivity; or one can start by saying that two objects, *a* and *b*, are identical only if some conditions Ψ are met. As Wittgenstein (2001: 5.5303) puts it, “to say of two things that they are identical is nonsense, and to say of one thing that it is identical with itself is to say nothing at all”. Wittgenstein (2001: 5.53, 5.533) himself thought that this is a sufficient reason to abandon the concept of identity altogether, and to eliminate the identity sign from conceptual notation. In the reformed language, according to him, the identity of an object would be expressed by the identity of its name rather than an identity statement. That means that no two objects would bare the same name, and no single object would bare two (or more) of them. Not too many philosophers followed Wittgenstein on that point (see Ramsey 1990 for an exception). For, even if one forms a language free of the identity sign, thus carrying no information about identity, to

square mathematical and ordinary language within it would come with a high price to pay. For most philosophers, then, the feeling remained that there is more to identity than a mere tautological description, that it goes beyond contradictory statements about it, and that it is a genuine phenomenon that needs to be explained, not eliminated. To cope with the feeling, one naturally turns to ways we *talk* about, or *express* identity in ordinary language, and then try to come up with a plausible explanation of the phenomenon based on the semantic analysis of relevant statements.

A clear example of such a strategy can be found in the opening passage of Frege's "On Sense and Reference." Frege (1960: 56) too starts with a metaphysical worry: "Equality gives rise to challenging questions which are not altogether easy to answer. Is it a relation? A relation between objects, or between names or signs of objects?" But, instead of offering a straightforward answer to these questions based on whatever considerations one would classify as metaphysical, Frege turns to considerations of identity *statements*. Afterwards, nowhere in his paper does he deal with the first question (although his second question strongly suggests the answer), and the second dilemma is settled explicitly only negatively: Neither option is acceptable to Frege because neither can explain the relevant phenomena concerning the identity statements. This Frege's point, I think, is based on the confusion of metaphysical and semantic (and epistemological) issues. The rest of Frege's paper deals exclusively with the latter issues, although the way he opens his paper, as well as the way he concludes it, suggests he deals with the former.

So, immediately after posing the questions about the nature of identity, Frege turns to consideration of identity statements. He distinguishes statements of the form " $a = a$ " (e.g. "Cicero is Cicero") from statements of the form " $a = b$ " (e.g. "Cicero is Tully"). The distinction, however, is not made on the ground that they have different *form* (see also Frege 1972: 124). Rather, Frege (1960: 56) insists, the distinction should be made because " $a = a$ " and " $a = b$ " differ in cognitive value: " $a = a$ holds *a priori* and, according to Kant, is to be labelled analytic, while statements of the form $a = b$ often contain very valuable extensions of our knowledge and cannot always be established *a priori*". This feature of identity statements—particularly statements of the form " $a = b$ "—Frege suggests, supports the view that identity is a relation between *names* of objects. To say that Cicero is Tully, for example, is to say that names "Cicero" and "Tully" designate the same object.

Given the general English conventions about functioning of proper names and the verb "is" (interpreted as identity)—namely, what a competent English speaker tacitly knows when correctly using these expressions—from the fact that the sentence "Cicero is Tully" is true there follows that names "Cicero" and "Tully" designate the same object. And it certainly comes as a discovery to learn that a person bears

another name, and what that other name is—a discovery one cannot know a priori unless he stipulatively introduced that other name into discourse.³ Accordingly, a statement expressing that circumstance cannot be analytic. I know, for example, that “Lady Gaga”, “Lil’ Kim”, and “Nicki Minaj”, are names of Lady Gaga, Lil’ Kim, and Nicki Minaj, respectively, and I strongly suspect that these are not the only names of these singers. But only after some googling, I discover what names they bear in addition, and that Lady Gaga is Stefani Germanotta, that Lil’ Kim is Kimberly Jones, and that Nicki Minaj is Onika Maraj. Given all that, the proposal that the difference in cognitive value be explained by appeal to metalinguistic information sounds appealing.

3.

Frege embraced the metalinguistic conception of identity, and for the similar reasons, in *Conceptual Notation* (Frege 1972: 124–126; 1960: 56). There, instead of the standard identity symbol “=”, Frege introduced a novel symbol, “≡”, that stands for the *identity of content* of symbols placed on the left and the right of it, and explained it as follows (Frege 1972: 124): “Identity of content differs from conditionality and negation by relating to names, not to contents. Although symbols are usually only representatives of their contents [...] they at once appear *in propria persona* as soon as they are combined by the symbol for identity of content, for this signifies the circumstance that the two names have the same content.”

Frege’s “≡” is more general in application than “=”. It can be combined with symbols with which “=”, strictly taken, cannot, as long as these symbols have a (conceptual) content.⁴ And, combined with the double judgment stroke, it serves to Frege as the indicator of abbreviative definition (Frege 1972: 126, 167–168).⁵ Other than that, there is no difference, and it would be wrong to conclude that Frege intended to use “≡” in addition to “=” (for the latter symbol is used nowhere in the concept script). Nor should one think that Frege intended to elimi-

³ For a discussion about the possibility of knowing a priori truths that are otherwise known *a posteriori*, see e.g. Kripke (1980: 63). Frege (1972: 167–168), and Russell and Whitehead (1927: 168), thought that such stipulative or abbreviative definitions are not identity statements on the par with “Cicero is Tully” or “Hesperus is Phosphorus”.

⁴ In his latter writings, Frege treated both singular terms and sentences as proper names of objects, so all such expressions could, from that perspective, flank the standard identity sign, and in his writings they do.

⁵ Russell and Whitehead in *Principia Mathematica* distinguished three senses of Frege’s “≡”—as identity, equivalence, and abbreviative definition—by representing them formally using different signs, namely, the identity sign “=” (1927: 22–23), the equivalence sign “≡” (1927: 7), and the definition sign “= Df”, which is to be taken as a single symbol, rather than as composed of two symbols, the identity sign and “Df” (1927: 11). Wittgenstein (2001: 4.241, 5.101) followed Russell and Whitehead in that respect.

nate “=” in any significant sense; certainly not in Wittgenstein’s (2001). The symbol “≡” is merely broader in application, and free of whatever unwanted burden “=” might bring for a reader into the concept script from mathematics and ordinary use, the burden which in *Conceptual Notation* Frege was eager to avoid. Therefore, wherever “=” would be used, “≡” could be used as well (but not vice versa). In its literal use, “Snow is white = Snijeg je bijel” would make little sense, but “Snow is white ≡ Snijeg je bijel” would be perfectly fine.⁶ And that there is a need for such a symbol, with the intended metalinguistic interpretation, Frege demonstrates using a geometrical example where “*A*” and “*B*” ultimately name the same fix point on the circumference of a circle around which a straight line rotates, and concludes (Frege 1972: 126) “that different names for the same content are not always merely an indifferent matter of form; but rather, if they are associated with different modes of determination, they concern the very heart of the matter. In this case, the judgement as to identity of content is, in Kant’s sense synthetic”. So, the point is the same as in the previously quoted passage—statements of the form “ $A \equiv B$ ”, just as the earlier ones of the form “ $a = b$ ” are always synthetic, and, at least sometimes known a posteriori.

Frege subsequently become dissatisfied with the proposed conception of identity. The problems he saw with it in “On Sense and Reference” are not fully clear, but it seems that his main point was that, if interpreted metalinguistically, “the sentence $a = b$ would no longer refer to the subject matter, but only to its mode of designation; we would express no proper knowledge by its means” (Frege 1960: 56). That most likely means that, e.g., the discovery that *Hesperus is Phosphorus* is not a linguistic discovery about the coreference of names “Hesperus” and “Phosphorus”. Rather, it is an astronomical discovery about a planet that goes beyond linguistic conventions of English. And the above proposed conception of identity apparently fails to capture that fact. So, even if the English sentence “Hesperus is Phosphorus” in some sense implies that names “Hesperus” and “Phosphorus” designate the same thing, it is certainly not what that sentence primarily, or literally, says. Apparently, then, Frege became dissatisfied with his early conception of identity for the same reason he was dissatisfied with formalist treatments of arithmetic. Just as numerals are not a proper subject-matter of arithmetic, so names are not a proper subject-matter of identity statements (see e.g. Frege 2013: ix). Russell and Whitehead (1927: 67) gave a similar objection to metalinguistic reading of identity statements. Their complaint was not, however, that such reading changes

⁶ An example of a nonstandard use of “=”, the one that appeals to our pragmatic intuitions, and which is defined nowhere in the book, can be found in Russell and Whitehead (1927: 138), where, for example, they interpret the proposition “*p*” as “*p* = Socrates is a Greek”, and the propositional function “*fx*” as “*fx* . = . *x* is a Greek”. These are certainly not identity statements, and, as it seems, they are not worthy of being labelled definitions.

the subject matter of identity statements, but rather their truth conditions, because part of truth conditions of any such statement would be that a certain object be called a certain name—but the truth of such a statement cannot depend on that feature.⁷

Of course, one can think of a number of other problems with the proposed conception of identity. The crucial one is that no matter how the thesis is ultimately spelled out, it will always presuppose the competing objectual conception of identity. Just consider Frege's two variants of metalinguistic definition of identity, namely "the symbol *A* and the symbol *B* have the same conceptual content" (Frege 1972: 126), and "the signs or names '*a*' and '*b*' designate the same thing" (Frege 1960: 56). Both definiens contain the phrase "the same", which must be interpreted in terms of the objectual identity relation. And an alternative metalinguistic definiens, namely, "the object named '*a*' is identical to the object named '*b*'" (Frege 1960: 78), faces the problem even more obviously. It follows, then, that any such metalinguistic definition of identity presupposes the objectual identity, and so, whatever it merits would be, it could not be its alternative, but, at best, a supplement. But, in the light of the above objection, should one even consider keeping the metalinguistic definition? One reason would certainly be to keep it not as the definition of identity, but rather as the explication of the content or truth conditions of identity statements. That would certainly be compatible with Frege's (1972, 1960) reasons to consider it in the first place. Nevertheless, one would still face Frege's initial objection.

The problem of presupposing the objectual identity could be avoided if the concept of identity occurring in definiens would be interpreted metalinguistically as well, but only at the cost of either the circularity of the definition or leading into the infinite regress. Russell most likely had that in mind when he noted that Frege's early take on identity is "a definition which, verbally at least, suffers from circularity" (1992: 502). Later, Frege made a related point when he wrote: "Since any definition is an identification, identity itself cannot be defined" (Frege 1984: 200). So, the problem of circularity would be double here: Not only does the definiens appeal to the very concept it should define, but the very definition of identity—whatever form it may take—is itself a case of identity statement, and as such it presupposes the concept. It is far from clear, however, that Frege's definition would be circular in the second sense. Namely, by defining the concept of identity of content, Frege is in fact not describing a previously established concept and determining the meaning of its familiar symbol. Rather, he introduces a novel symbol and stipulates its meaning, thus bringing a new concept. And, given the way he understood such stipulative definitions, it is far from clear that they are the case of identity statements. Also, to what

⁷ They originally made that point for cases with definite descriptions, but the point goes for other singular terms as well. For a similar objection see Kripke (1980: 108).

degree Frege's concept overlaps with the familiar concept of identity might—with respect to the problem of circularity—be irrelevant. I will return to that issue in section 5.

If identity in general cannot be properly defined, since every definition of identity has the form of identity statement, and if, in addition, identity cannot be a relation between names, since that would commit us either to accept the objectual identity, or it would lead us into circularity and infinite regress, it seems that one has no choice but to grant that identity is an undefinable relation between objects. But that option Frege found equally unsatisfied. For him, the same thing that supports the metalinguistic conception—namely, the informativeness of identity statements of the form " $a = b$ "—undermines the objectual conception. If identity would merely be a relation in which every object stands to itself and no other object, " $a = a$ " and " $a = b$ " would say the same thing, and would thus differ only in form. But that is obviously not the case. For Frege, there is more to identity than that.

4.

The conclusion of the opening passage of "On Sense and Reference" is that neither of the two mentioned options is the acceptable one. And, as far as identity goes, the concept is further discussed nowhere in the paper. The ultimate conclusion of the passage is only that "a difference [between statements ' $a = a$ ' and ' $a = b$ '] can arise only if the difference between the signs [a ' and ' b '] corresponds to a difference in the mode of presentation of that which is designated". *This* tells us nothing about identity itself. And the rest of the paper is merely an elaboration and extension of this conclusion. Indeed, Frege's closing passage in the paper seems misleading on that matter. He writes:

Let us return to our starting point. / When we found ' $a = a$ ' and ' $a = b$ ' to have different cognitive values, the explanation is that for the purpose of knowledge, the sense of the sentence, viz., the thought expressed by it, is no less relevant than its reference, i.e. its truth value. If now $a = b$, then indeed the reference of ' b ' is the same as that of ' a ,' and hence the truth value of ' $a = b$ ' is the same as that of ' $a = a$.' In spite of this, the sense of ' b ' may differ from that of ' a ,' and thereby the thought expressed in ' $a = b$ ' differs from that of ' $a = a$.' In that case the two sentences do not have the same cognitive value. (Frege 1960: 78)

The passage is misleading because Frege's starting point was the question about *identity*, not *identity statements*, and these are two different, although related things. Frege does not provide any metaphysical view about identity in spite of his initial metaphysical question. Instead, he offers a semantic analysis of identity statements, based on the sense/reference distinction.

To make an analogy: It is one thing to ask, for example, do propositional attitudes exist, and, if so, are they relations, what (if anything) they relate, etc. These are metaphysical issues. It is quite another thing

to ask what propositional attitude *reports*, namely sentences reporting subject's particular attitude, typically say, what are their truth conditions, etc. The metaphysics of propositional attitudes one embraces at the outset might help in forming the semantic analysis of attitude reports with which one will ultimately end up. Just as it might turn out that the semantic analysis of attitude reports one embraces will ultimately determine the way one understands attitudes themselves. Nevertheless, these are two different issues that should not be conflated. The same goes for identity and identity statements (and virtually any other metaphysical issue that finds its counterpart in semantic discussions concerning the accompanying vocabular; just think of universals or time).

In addition, the last quoted passage contains another problematic point. Recall, in the opening passage of the paper, we are left only with the negative answer to the question whether identity is a relation between objects or between names of objects. And now, given the intonation, it seems that Frege is at least hinting which of the two options he accepts when he writes “[i]f now $a = b$, then indeed the reference of ‘ b ’ is the same as that of ‘ a ’”. But what option is that? On the closer inspection, one finds that this formulation is ambiguous, and that it is compatible with either of the two options, since, on the par with Frege’s (1980a: 69) transformation of numerical statements of the form “ x has N ps ” into identity statements of the form “the number of x ’s ps is (identical to) N ”, one could transform Frege’s two formulations of the metalinguistic definiens, namely, “the symbol A and the symbol B have the same conceptual content” (Frege 1972: 126), and “the signs or names ‘ a ’ and ‘ b ’ designate the same thing” (Frege 1960: 56), into formulations resembling the above ones, namely: “the conceptual content of the symbol A is the same as that of B ” and “the thing designated by the sign of name ‘ a ’ is the same as that of ‘ b ’”. In fact, Frege in *Conceptual Notation* at one point, reflecting on his geometrical example demonstrating the informativeness of identity statements, writes that “the name B has the same content as the name A ” (1972: 125).

Nevertheless, given the way Frege appeals to identity in *The Foundations of Arithmetic* and his other writings after *Conceptual Notation*, one could have little doubt about which concept of identity he embraces. It is the plain objectual concept according to which identity is the relation in which an object stands to itself and no other object.⁸ For him to

⁸ Frege slipped into the metalinguistic interpretation even after *Conceptual Notation*: “[...] ‘the number of Jupiter’s moons is the number four, or 4’. Here ‘is’ has the sense of ‘is identical with’ or ‘is the same as’. So that what we have is an identity, stating that the expression ‘the number of Jupiter’s moons’ signifies the same object as the word ‘four’.” (Frege 1980a: 69). Apparently, such an interpretation comes naturally. Similarly, Kripke, a clear opponent of the metalinguistic interpretation (see Kripke 1980: 107–108), at one point in his book writes that “sometimes we may discover that two names have the same referent, and *express this* by an identity statement” (Kripke 1980: 28, my italics). Pace Kripke, we would more likely express

embrace the alternative formulation would be redundant, since in his later writings the phenomenon of informativeness is explained by appealing to senses, rather than modes of designation that he introduced with his early conception of identity. So, given the way the argumentation in the initial passage of “On Sense and Reference” is set, Frege should not have objected to the objectual view of identity that it cannot account for the alleged difference in cognitive value, because the view was never intended to be such an explanation. Instead, he should have said that although *identity* is a relation “in which each thing stands to itself but to no other thing” (Frege 1960: 56), *identity statements*, at least those of the form “ $a = b$ ”, convey information that goes beyond that metaphysical dictum; hence the difference in thoughts expressed by “ $a = a$ ” and “ $a = b$ ”. Keeping that in mind, the issue Frege is particularly concerned with is whether the information identity statements involve is: (a) information about names flanking the identity sign, and their semantic conventions—ways of designation; (b) information about the object in question that is given in different ways, independently of the way it is designated; or (c) merely the information about the self-identity of an object.

All three options, and not just (c), plainly presuppose the objectual view of identity. Indeed, one would think, it seems impossible to sidestep the objectual view since it is incorporated into the very way we think about objects and the way they are related. So, is there more to be said about identity?

5.

Frege and Russell in principle agreed on many points concerning the concept of identity. For one thing, both of them appealed to identity statements and their informativeness to point out the need for a semantic analysis that goes beyond mere reference of relevant expressions. Frege thought that it strongly supports his sense/reference distinction, Russell (1992: 63) took it as crucial for the semantics of descriptive phrases (see also Russell and Whitehead 1927: 23). Also, both of them accepted Leibniz’s indiscernibility of identicals principle as the fundamental law governing identity, perhaps even its definition. Thus, in *The Foundations of Arithmetic*, Frege (1980a: 76) writes: “Now Leibniz’s definition is as follows: ‘Things are the same as each other, of which one can be substituted for the other without loss of truth’. This I propose to adopt as my own definition of identity”. In *Conceptual Notation* (Frege 1972: 161–162), and later in *Basic Laws of Arithmetic* (Frege 2013: 36), Frege introduced variants of the principle as one of the axioms (or basic laws) of the logical system, and Russell and Whitehead (1927: 23) write: “If x and y are identical, either can

this with the statement “ a ’ and b ’ are coreferential”. “ a ” and “ b ” would not be used, but only mentioned.

replace the other in any proposition without altering the truth-value of the proposition.”

Taken at face value, the principle of indiscernibility of identicals, be it a definition or merely “a principle that brings out the nature of the relation of identity” (Frege 1984: 200), or “a fundamental property of identity, from which remaining properties mostly follow” (Russell and Whitehead 1927: 23), it makes no sense. Firstly, the plural “things”, the phrase “each other”, and similar, are in obvious conflict with the very idea of identity, for no *two* (or more) things could ever be identical with each other. Secondly, even if this awkward wording is ignored, the idea that *things* are substituted is just as bad: Where exactly would we substitute a thing, and, for any given thing, for what other thing should it be substituted, and truth of what could be lost? A way to make some sense from at least a part of that formulation would be to say that propositions, rather than sentences, are primary truth bearers, that objects are their constituents, and so that one substitutes objects within propositions. The problem with that would obviously be that whenever identity holds for whatever object—and, by definition, it always holds for every object—no *other* object could be substituted for it on the ground of identity. Thus, if anything is substituted in such cases, it is certainly not an object entering the identity relation.

If one is to make any sense of Leibniz’s indiscernibility of identicals principle, at least a fundamental revision of its formulation is needed. Frege did not address this issue explicitly, but the way he immediately utilised it, suggests that he most likely was aware that in its original form it makes no sense. Thus, in the same paragraph, Frege writes:

Now, it is actually the case that in universal substitutivity all the laws of identity are contained. / In order, therefore, to justify our proposed definition of the direction of a line, we should have to show that it is possible, if line *a* is parallel to line *b*, to substitute “the direction of *b*” everywhere for “the direction of *a*”. (Frege 1980a: 77)

Similarly, Russell and Whitehead (1927: 23) start with the formulation: “[i]f *x* and *y* are identical, either can replace the other [...]”, but just a passage below continue: “[identity] can only hold between *x* and *y* if *x* and *y* are different symbols for the same object”. They clearly talk first about identity as an objectual relation—since, taken as objects in their own right, “*x*” and “*y*” are definitively not identical—and symbols “*x*” and “*y*” are *used* to represent objects.⁹ But then they switch to metalinguistic mode, *mentioning* these symbols in the passage that follows. Taken in conjunction, the quoted section makes little sense. And they obviously did not intend to accept the metalinguistic conception, since, a bit further, they explicitly criticise it (Russell and Whitehead 1927: 67).

⁹ Throughout *Principia Mathematica* one finds a number of places that support that reading, e.g., in their phrase “the objects which are identical with *x*”. Here, “*x*” stands for an object, and it is not a disguised name of the symbol “*x*”.

If Leibniz's definition is to be interpreted (or rephrased) in the light of the quoted passages, it is clear that it is not a thing that is substituted, but rather its name, that it is substituted for another name it bears, and that the substitution takes place in a sentence. In that case, the truth of a sentence in which it occurs is preserved.¹⁰ So the definition would now be: *a* is identical to *b* only if *a*'s name "*a*" can be substituted for (its other) name "*b*" in a sentence without the loss of its truth. But this could hardly be taken as a definition of *identity*; if it were, what would it tell us about it? Rather, we already have to possess the concept of identity to make sense of such a formulation. Subsequently, Frege become dissatisfied with Leibniz's definition of identity because it, or any other definition of identity, would be circular (Frege 1984: 200): "Leibniz's explanation [...] does not deserve to be called a definition [...] Since any definition is an identification, identity itself cannot be defined".

6.

Now, if we consider this Frege's remark in the light of his earlier distinction between plain identity statements and abbreviative definitions, he obviously thinks that identity is a concept we already possess (see also Frege 1980a: 74), and can subsequently only describe. It is not a concept we introduced by a definition. Thus, a definition of identity would itself be an identity statement on the par with "Cicero is Tully". Russell and Whitehead (1927: 11, 57) took a different course, writing as if identity—at least in the context of their formal system—is introduced stipulatively, and thus that its very definition is not an identity statement. Its definition, just as any other definition in *Principia Mathematica*, according to them, would be normative—"the expression of a volition"—rather than descriptive; it "is concerned wholly with the symbols, not with what they symbolise"; and it is neither true nor false, since it is not asserted (1927: 11). But, as far as the definition of identity goes, their view seems problematic, since it is far from clear that the definition of identity is such a definition. If that is so, one should side with Frege's ultimate conclusion, namely, accept that no definition of identity is possible. At best, one could end up with its "informative analysis" that would explicate its features. Pace Wittgenstein (2001), it should be observed that, even if one could build a formal language devoid of the identity sign, one could hardly square ordinary and mathematical language within it. And if the latter ones are the phenomena one should explain, rather than explain away, a sufficiently strong formal system for that purpose should certainly keep the identity sign with its preestablished use, and the concept of identity lurking behind it.

¹⁰ For the sake of simplicity, I ignore here the issue of intensional sentences, i.e., the problem some such sentences pose for unrestricted substitutivity.

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